Mathematics Competition for the
Seventh Graders of Helsinki 2013/2/7

- The time allotted is 50 minutes.

- The allowed tools are writing and drawing instruments, i.e. pencil, eraser, ruler and compass. Calculators and mathematical tables are not allowed.

- Each problem is worth one point. Wrong answers are not punished.

- The problems are not ordered in increasing difficulty, but the first problems are likely to be easier than the last ones.

1. A bookstore sells pocket books nine euros apiece, and they have started a marketing campaign during which after buying seven pocket books one gets an eight for free. A friend of books finds 56 interesting pocket books in the bookstore. How much do they cost?

a) 392 €   **b)** 432 €   **c)** 441 €   **d)** 495 €   **e)** 504 €

2. The measures of the floor of a room are 3×5 metres, and its height is 3 metres. Tiling the floor requires 60 tiles. How many tiles are needed to tile the entire room (including the walls, the floor and the ceiling)?

a) 240   **b)** 312   **c)** 360   **d)** 372   **e)** 390

3. In a certain country, one inhabitant consumes 12 kg of coffee per year in average. How many tons of coffee is consumed in the country per year, if its population is 5,3 million people?

a) less than 10 tons
b) more than 10 and less than 100 tons
c) more than 100 and less than 1000 tons
d) more than 1000 and less than 10000 tons
e) more than 10000 tons

4. What is

$$\underbrace{2012 + 2012 + 2012 + \ldots + 2012}_{\text{2012 appears here 2013 times}} - \underbrace{2013 - 2013 - 2013 - \ldots - 2013}_{\text{2013 appears here 2012 times}} ?$$

a) -4025 **b)** -2013 **c)** 0   **d)** 1   **e)** 2012

5. A small child has four building blocks of different colors, and has firmly decided to build from them a tower four blocks high. In how many different orders can colors appear in the tower?

a) 10   **b)** 12   **c)** 16   **d)** 18   **e)** 24

6. It is known about two integers x and y, that they are both odd and that their difference is seven. How many such pairs of numbers are there?

a) none   **b)** one   **c)** five   **d)** more than a hundred, but less than a thousand
**e)** infinitely many

7. Maija is traveling by train. The train is moving at the speed of 180 km/h when she sees a water tower in front from an angle of 45° from the railway track. In order to find out how far away the tower is from the track, she decides to start the timer of her cellphone, and after ten seconds she sees the tower at behind, again from a 45° angle from the track. How far away is the tower approximately from the track?

      a) 250 m   **b)** 350 m   **c)** 500 m   **d)** 700 m
      e) the given data are not enough to answer the question

8. Of five numbers the first is 1 and the last is 9. Furthermore, the product of any three consecutive numbers in the list is 3. What is the number in the middle of the list?

      a) $\frac{1}{3}$ **b)** $\frac{1}{\sqrt{3}}$ **c)** 1   **d)** $\sqrt{3}$ **e)** 3
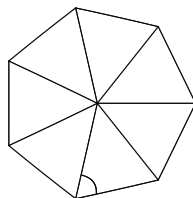
9. A triangle is drawn inside a square so that the vertices of the triangle are on the sides of the square
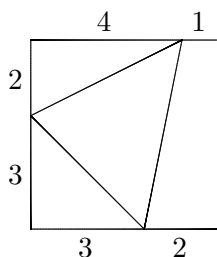
What is the area of the triangle?

      a) 14   **b)** 15   **c)** 16   **d)** less than 14   **e)** more than 16

10. In the following picture there is a regular 7-gon (i.e. a "heptagon"), and its vertices have been joined to its center using straight line segments. How large is the angle marked into the picture?

      a) 50°   **b)** $51\frac{3}{7}°$ **c)** 60°   **d)** $64\frac{2}{7}°$ **e)** $72\frac{4}{7}°$

11. If A is a number such that $A^2 + A + 1 = 0$, then what is $\frac{1}{A^2}$?

      a) 1   **b)** A **c)** A^2 **d)** 0   **e)** -1

12. The measures of a sheet of paper are 24×32. It is folded so that one corner comes to lie on the opposite corner. How long is the crease (i.e. the line segment created by the folding)?

      a) 26   **b)** 28   **c)** 30   **d)** 32   **e)** 34

13. The numbers

$$0^2, \quad 1^2, \quad 2^2, \quad 3^2, \quad 4^2, \quad \ldots, \qquad \text{i.e.} \qquad 0, \quad 1, \quad 4, \quad 9, \quad 16, \quad \ldots,$$

are called square numbers. When a square number is divided by five, what are the possible remainders?

      a) 0 and 1   **b)** 0, 1 and 2   **c)** 0, 1 and 4   **d)** 0, 1, 3 and 4   **e)** 0, 1, 2, 3 and 4

14. In how many ways can we choose positive integers x and y so that $x^4 + y = 10001$?

      a) in one way   **d)** in infinitely many different ways
      b) in 10 different ways   **e)** not possible
      c) in 100 different ways