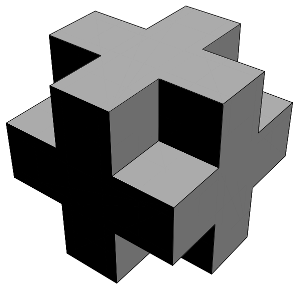
7th graders math contest, Turku
Final 23.4.2016

1. We saw off a small $1 \times 1 \times 1$ cube from each corner of a bigger $3 \times 3 \times 3$ cube. Find the area and the volume of the remaining object. A picture of this object is below



2. The notation n^3 means the third power $n \cdot n \cdot n$ of the number n. For example $5^3 = 5 \cdot 5 \cdot 5 = 125$. We tell you that

$$1^3 + 2^3 + 3^3 + 4^3 + \ldots + 10^3 = 3025,$$

and

$$1^3 + 2^3 + 3^3 + 4^3 + \ldots + 20^3 = 44100.$$

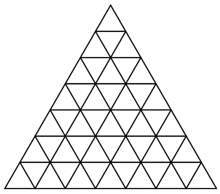
What is the following sum

$$1^3 + 3^3 + 5^3 + 7^3 + \ldots + 19^3?$$

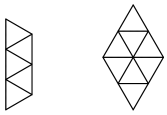
3. Every Tuesday Matti, Heta, Juha, Oona, Reetta and Teemu meet for a math club. The students gather around a table with six chairs. Matti always sits in the same place. Heta wants to sit next to Matti. Oona and Reetta don't get along well, and thus they don't want to sit next to each other. How many possible ways are there for the students to sit around the table?

4. Aleksi has a large supply of both black and white marbles. He lines them up observing the following rules. Every marble must have the same color as the marble five positions after it (if such a marble exists – if we reach the end of line before we get to the fifth, then this rule does not apply). Similarly, each marble must have the same color as the marble seven positions before it (if such a marble exists). So if he is using marbles of one color only, he will automatically obey both rules. But can Aleksi make as long two-colored lines as he wishes? If he can, tell him how to do it! If he cannot, then help him to build as long a two-colored line as possible.

5. Let us consider the following figure consisting of equilateral triangles.



Is it possible to cover this large triangle with tiles of the following types



As is always the case in tiling problems, you are not allowed to leave any gaps. The tiles may not overlap each other, and they cannot cross the boundaries of the large triangle.