# SEVEnth Graders' <br> Mathematics Competition final, 24 April 2019 

1. What is the last digit of the number $2015 \cdot 2016 \cdot 2017 \cdot 2018 \cdot 2019$ ?
2. In a triangle $\triangle A B C$ we have $\angle B A C=27^{\circ}$. From its side $A C$ a point $P$ has been chosen so that $\angle P B A=20^{\circ}$, and from its side $A B$ another point $Q$ has been chosen so that $\angle A C Q=19^{\circ}$. The line segments $B P$ and $C Q$ intersect each other at the point $X$. How large is the angle $\alpha$ (that is the angle $\angle P X Q$ ) marked in the picture?

3. A mathematician has a bookshelf, and on it three mathematics books in Finnish, four in English and two in Swedish. In how many ways can she order her books on the shelf, given that she wishes that any two books in the same language must be next to each other?
4. Prove that it is possible that among five people there is no set of three people such that all three know each other or such that each of them is unknown to both of the other two. (Here knowing is assumed to be symmetric in the sense that if Martha knows Saul, then Saul knows Martha.)
5. Which of the numbers

$$
2^{\left(3^{4}\right)}, \quad 2^{\left(4^{3}\right)}, \quad 3^{\left(2^{4}\right)}, \quad 3^{\left(4^{2}\right)}, \quad 4^{\left(2^{3}\right)}, \quad \text { and } \quad 4^{\left(3^{2}\right)}
$$

is the largest? [When $x$ is a number and $k$ is a positive integer, the notation $x^{k}$ means the product $x \cdot x \cdot x \cdot \ldots \cdot x$, where $x$ appears $k$ times. For example, $7^{5}=7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$ and $2^{\left(2^{3}\right)}=2^{2 \cdot 2 \cdot 2}=2^{8}=2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2=256$.]

