Mathematics Competition for the Seventh graders of Oulu Sub-Region, 17–21 February 2020 Solutions

1. Compute $100 - (30 - 5) - 25$.

a) 40 **b)** 50 **c)** 65 **d)** 90 **e)** 100

Solution. b) 50:

$$100 - (30 - 5) - 25 = 100 - 25 - 25 = 75 - 25 = 50$$

2. Compute $1 - 20 + 2 - 19 + 3 - 18 + \ldots + 19 - 2 + 20 - 1$.

a) -15 **b)** -1 **c)** 0 **d)** 15 **e)** 420

Solution. c) 0: By reordering the terms of the sum we get

$$\begin{aligned}&1 - 20 + 2 - 19 + 3 - 18 + \ldots + 19 - 2 + 20 - 1\\ =&(1 - 1) + (2 - 2) + (3 - 3) + \ldots + (19 - 19) + (20 - 20) = 0\end{aligned}$$

3. Integers are picked randomly between 1 and 20. At least how many numbers must be picked so that at least one of the picked numbers is divisible by three?

a) 3 **b)** 6 **c)** 10 **d)** 15 **e)** 20

Solution. a) 15: There are 6 integers divisible by three between 1 and 20 and 14 integers that are not divisible by three. Hence at least 15 numbers must be picked so one can be certain that at least one of the picked numbers is divisible by three.

4. The recipe of a pie includes 200g of sour cream and 3dl of berries. One gets 12 slices from one pie. A football club organizes a fair and they try to make as many pies as possible for sale. They have in their use 2,4kg of sour cream and 10 litres of berries. How many slices of pie can they make for sale at most? (Take into account that if they run out of either sour cream or berries they have to stop baking.)

a) 144 **b)** 100 **c)** 12 **d)** 360 **e)** 400.

Solution. a) 144: There is enough sour cream for 12 pies and berries for over 30 pies so they run out of sour cream first. Hence they can get at most $12 \cdot 12 = 144$ slices for sale.

5. Which of the following numbers is the sum of four consecutive positive integers?

a) 20 **b)** 21 **c)** 22 **d)** 23 **e)** 24

Solution. c) 22: The sum of four consecutive positive integers is of the form

$$n + (n + 1) + (n + 2) + (n + 3) = 4n + 6.$$

When $n = 4$ we get $4 \cdot 4 + 6 = 22$.

6. If $a \star b = a \cdot b + 3$ what is $3 \star 4$?

a) 7 **b)** 10 **c)** 12 **d)** 15 **e)** 21

Solution. d) 15: $3 \star 4 = 3 \cdot 4 + 3 = 15$.

7. There are 12 balls in the blue basket and the red basket together, 15 balls in the blue basket and the yellow basket together and 7 balls in the yellow basket and the red basket together. How many balls are there in the red basket?

a) 0 **b)** 2 **c)** 4 **d)** 5 **e)** The problem cannot be solved with the given information.

Solution. b) 2: Since there are 12 balls in the blue basket and the red basket together and 15 balls in the blue basket and the yellow basket together, there are 3 balls more in the yellow basket than in the red basket. Since there are 7 balls in the yellow basket and the red basket together, there are 2 balls in the red basket (and 10 in the blue basket and 5 in the yellow basket).

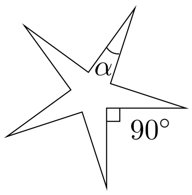
8. How many positive integers m satisfy the inequality

$$m \cdot (7 - m) > 0?$$

a) 0 **b)** 6 **c)** 7 **d)** 8 **e)** infinitely many.

Solution. b) 6: Since m is positive then $m(7-m)$ is positive when $7 - m > 0$, that is when $7 > m$. This is satisfied by the positive integers $m = 1, 2, 3, 4, 5, 6$.

9. How large is angle α when all the points of the star are the same size and all the angles between the points of the star are 90°?



a) 9° **b)** 18° **c)** 27° **d)** 36° **e)** 72°

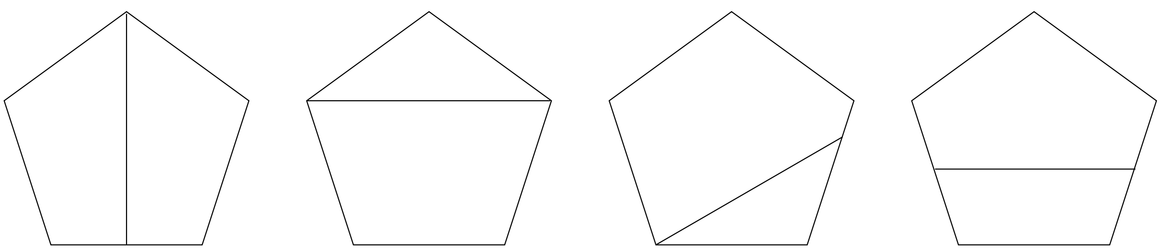
Solution. b) 18°: The star in the picture is a decagon so the sum of its inner angles is $8 \cdot 180°$. The larger inner angles are $360° - 90° = 270°$ each, so the size of α is

$$\frac{8 \cdot 180° - 5 \cdot 270°}{5} = 8 \cdot 36° - 270° = 18°.$$

10. A regular pentagon is divided into two parts by a line. Which of the following cases is NOT a possible combination for the shapes of the two parts?

a) Two quadrangles **b)** Two pentagons **c)** A triangle and a quadrangle
d) A triangle and a pentagon **e)** A quadrangle and a pentagon

Solution. b) When a pentagon is divided into to parts, at most 4 new angles are formed inside the pentagon so there are at most $5 + 4 = 9$ angles in the two parts together. Since there are 10 angles in two pentagons together, option b) is not possible. The other options are possible as can be seen from the picture:



11. A sequence consists of 2020 numbers each of which is either 1 or -1. The same number may appear in the sequence at most three times in a row. What is the biggest possible value of the sum of all the numbers in the sequence?

a) 0 **b)** 505 **c)** 1010 **d)** 1515 **e)** 2020

Solution. c) 1010: The sum of all the numbers in the sequence is biggest when there are as many ones in the sequence as possible. Since the same number may appear in the sequence at most three times in a row, then after three ones there must be a -1. The greatest sum is achieved by a sequence that is constructed by repeating the pattern 1,1,1,-1. Since $2020/4 = 505$ there are 505 of these patterns in the sequence. Each of the patterns increases the sum by two so the greatest sum is $2 \cdot 505 = 1010$.

12. Joonas and Jussi both have 100 euros of cash. On the first day Joonas deposits one tenth of his cash into his bank account while Jussi withdraws the amount corresponding to one tenth of his cash from his own bank account. On the second day Jussi deposits one tenth of his cash into his bank account while Joonas withdraws the amount corresponding to one tenth of his cash from his own bank account. Which one has more cash by the end of the second day?

a) Jussi **b)** Joonas **c)** They both have the same amount of cash **d)** The answer depends on the amount of money on Jussi's account **e)** The answer depends on the amount of money on Joonas's account

Solution. c) They both have $100 \cdot \frac{11}{10} \cdot \frac{9}{10} = 100 \cdot \frac{9}{10} \cdot \frac{11}{10}$ euros.

13. How many different 4-letter words can you get from letters A, B, C, A? (The words don't have to mean anything.)

a) 6 **b)** 12 **c)** 18 **d)** 24 **e)** 30.

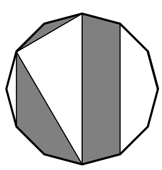
Solution. b) 12: If the first letter of the word is A, then there are $3 \cdot 2 \cdot 1 = 6$ possible orders for the last three letters. If the first letter is B or C, then there are three possible orders for the last three letters. Hence there are $6 + 3 + 3 = 12$ words in all.

14. Two frogs, Samu ja Panu, will leap across a track of 60 cm. All of Panu's leaps are of equal length. Samu's first leap is 2 cm and the following leaps are always as long as the distance Samu has covered so far. How long must Panu's leaps be at least so that both frogs get to the end of the track with the same amount of leaps?

a) 5 cm **b)** 8 cm **c)** 10 cm **d)** 15 cm **e)** 20 cm

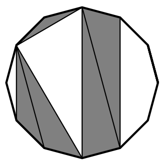
Solution. c) After n leaps Samu has covered the distance of 2^n cm. Since $2^5 = 32$ and $2^6 = 64$, Samu will get to the end of the track with 6 leaps. Panu's leaps must be at least $60/6 = 10$ cm and at most $60/5 = 12$ cm for it to get to the end of the track with 6 leaps.

15. Below is a picture of a regular 12-gon. If the area of the 12-gon equals to 1, what is the area of the gray part?

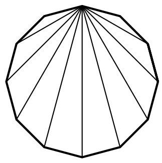


a) $\frac{1}{5}$ **b)** $\frac{1}{4}$ **c)** $\frac{1}{3}$ **d)** $\frac{5}{12}$ **e)** $\frac{1}{2}$

Solution. The gray part can be divided into triangles as follows:



On the other hand we can divide the 12-gon into triangles:



In this picture there are five kinds of triangles, two of each. In the gray part there is exactly one triangle of each type. Hence the area of the gray part is on half of the area of the whole 12-gon which is 1/2.